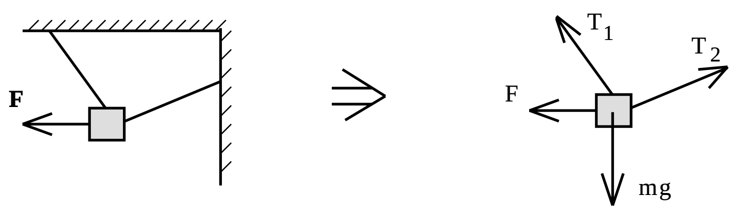
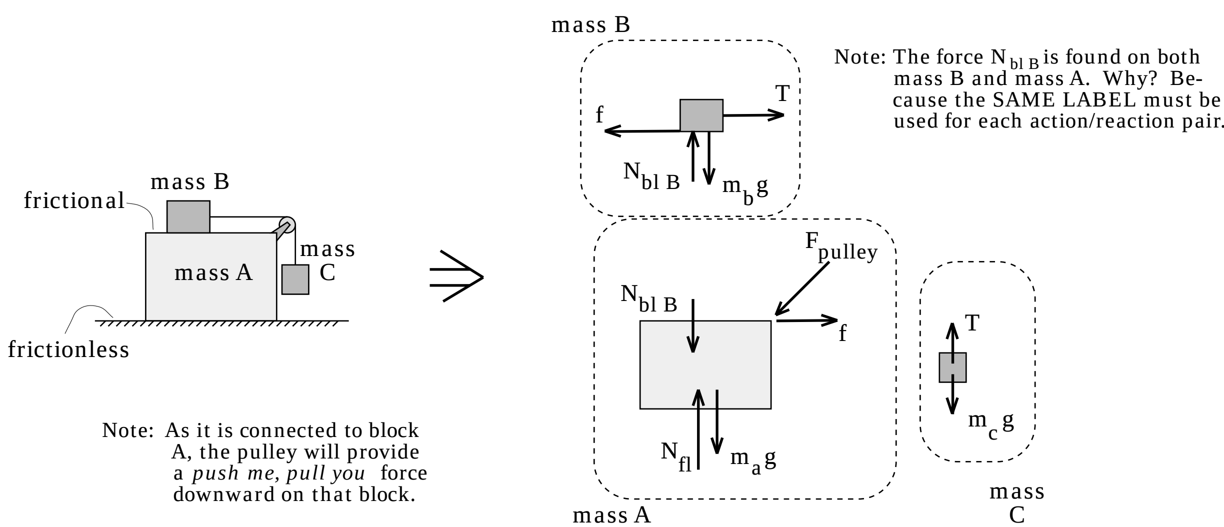
CHAPTER 5 -- NEWTON'S LAWS

5.1) Drawing a *free body diagram* for the force of EACH BODY in each sketch:

a.)



b.)

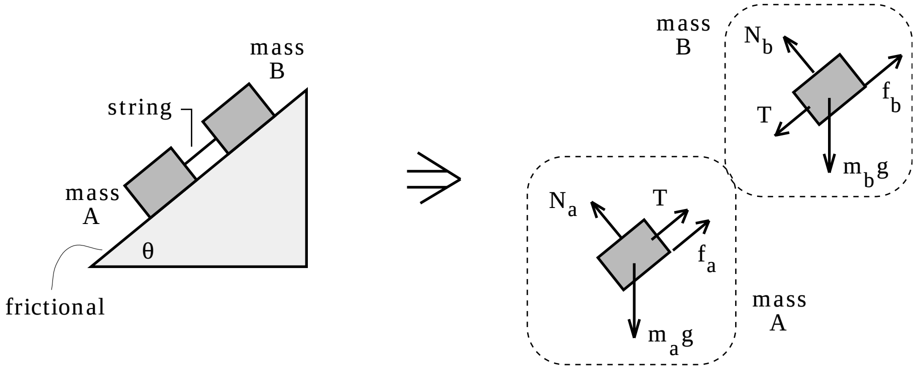


Note 1: There are two *action/reaction force pairs* between *masses A* and *B*: the normal force $N_{bl\ B}$ that *A* applies to *B* and vice versa, and the frictional force *f* between the two. Be sure you understand what is going on here!

Note 2: Notice that the *magnitude* of the tension force *T* on *mass C* and *mass B* is the same.

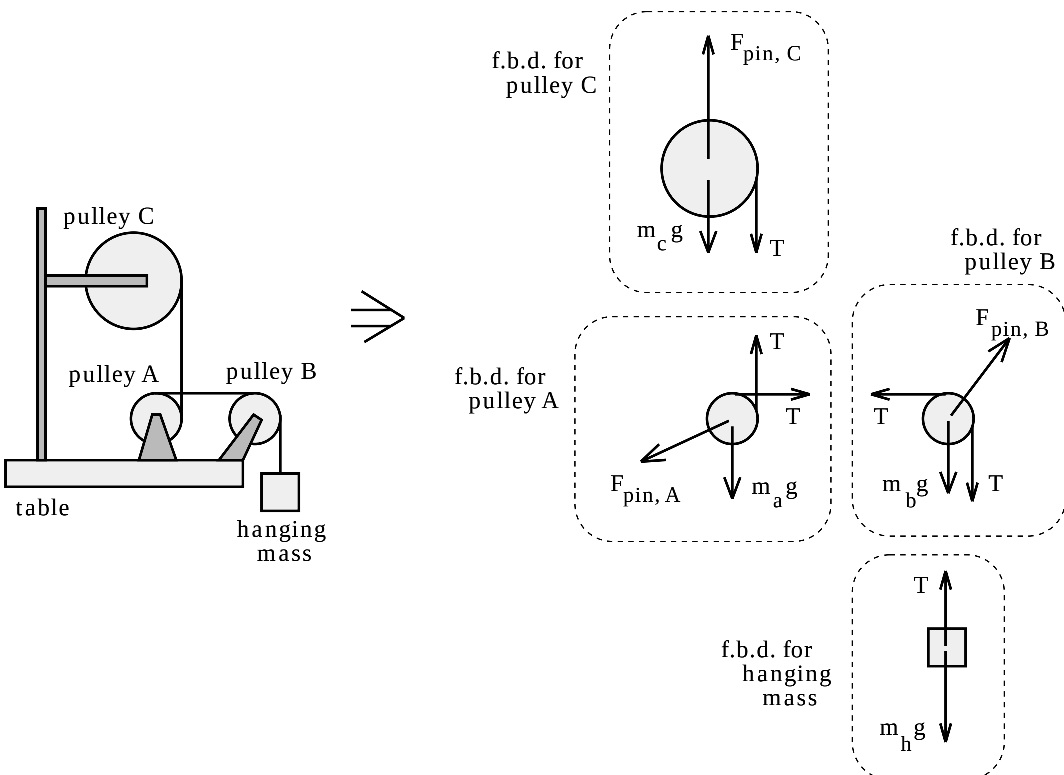
Note 3: The pulley mount on *mass A* applies a *downward and to the left* force F_{pulley} on *mass A*. As we are interested in ALL the forces acting on each mass, that force has to be included.

c.)



d.)
Note 1: All the pulleys do here is re-direct the line of the tension T.

Note 2: The pin that holds each pulley in place must exert a force that effectively keeps its pulley from flying off into space.



Note 3: There is a force acting at *the pin of each pulley* to keep the pulleys from falling through the table.

5.2) According to Newton's Third Law:

a.) The reaction to *the force the floor applies to you* is *the force you apply to the floor*.

b.) The reaction to *the force a string applies to a weight* is *the force the weight applies to the string*.

c.) The reaction to *the force a car applies to a tree* is *the force the tree applies to the car*.

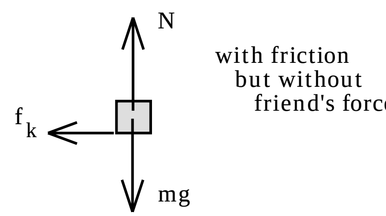
d.) The reaction to *the force the earth applies to the moon* is *the force the moon applies to the earth*.

5.3)

a.) A *free body diagram* for the situation *before your friend applies his force* (*Part B*) is shown below. Making a_x into a MAGNITUDE by unembedding the negative sign, N.S.L. yields:

$\underline{\sum F_x:}$

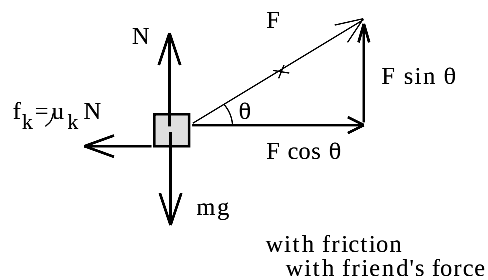
$$- f_k = - ma_x$$
$$\Rightarrow \quad - (12 \text{ nt}) = - (30 \text{ kg})\, a$$
$$\Rightarrow \quad a = .4 \text{ m/s}^2.$$



Note 1: Why make a_x into a magnitude by unembedding the negative sign? In certain kinds of problems, doing so will make life easier. Get used to it.

Note 2: In the next question, you are going to need μ_k. From the f.b.d. above, $N = mg = (30\ kg)(9.8\ m/s^2) = 294\ nts$. As $f_k = \mu_k N$, we can write $\mu_k = f_k/N = (12\ nt)/(294\ nt) = .04$.

b.) With the additional force applied by your friend, the *free body diagram* looks like the one shown to the right (note that *N* has changed). To determine *N*:



$\underline{\sum F_y:}$

$$N + F \sin 40^o - mg = - ma_y \qquad (= 0 \text{ as } a_y = 0)$$
$$\Rightarrow \quad N = -F \sin 40^o + mg$$
$$= - (60 \text{ nt}) \sin 40^o + (30 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= 255 \text{ nts.}$$

$\underline{\sum F_x}$:

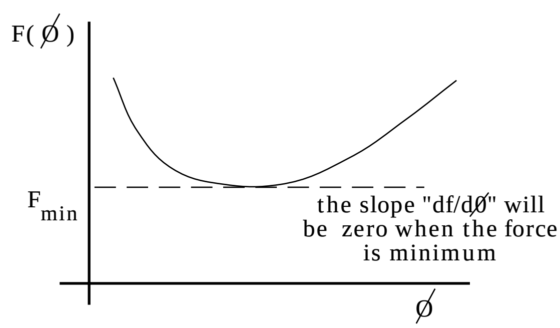
$$-\mu_k N + F\cos 40^o = -ma_x.$$

Note 1: In this case, I have assumed that your friend's force will not overcome that of friction and the *direction* of the sled's *acceleration* will still be negative (i.e., to the left). As such, I have unembedded the negative sign in front of the *ma* term. If I am wrong, the SIGN of the calculated acceleration will be negative. Continuing:

$$-\mu_k N + F\cos 40^o = -ma_x$$
$$\Rightarrow -(.04)(255\text{ nt}) + (60\text{ nt})(.766) = -(30\text{ kg})\,a$$
$$\Rightarrow \qquad a = -1.19\text{ m/s}^2.$$

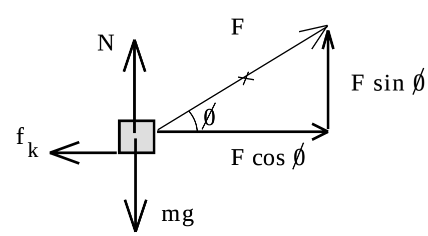
Note 2: The negative sign means that I've assumed the wrong direction for *a*. Evidently, your friend's force was greater than the frictional force and the acceleration was really in the *+x direction* (if this ever happens to you, what I've just said is all you will have to state to make the problem OK).

c.) The graph of $F(\phi)$ *vs.* ϕ looks something like the graph shown in the figure to the right. Notice that at the minimum, the slope of $F(\phi)$ is ZERO (that is, $dF(\phi)/d\phi = 0$ at that point). All we have to do is generate an expression for the force as a function of ϕ, then put its *derivative* equal to zero and solve for ϕ under that condition. That will produce the angle at which the force is a minimum.



Executing that operation:

i.) The f.b.d for the situation is shown to the right.



ii.) Noting that we will need to use $\mu_k N$ for the *frictional force* f_k, we will start with N.S.L. in the *y direction* to determine *N*:

$\underline{\sum F_y:}$

$$N + F\sin\phi - mg = ma_y$$
$$= 0 \qquad (\text{as } a_y = 0)$$
$$\Rightarrow \quad N = -F\sin\phi + mg \qquad \text{(Equation A).}$$

iii.) Using N.S.L. in the *x direction*, we get:

$\underline{\sum F_x:}$

$$-\mu_k N + F\cos\phi = ma_x$$
$$= 0 \ \ (\text{as the velocity is constant}).$$

iv.) Substituting *Equation A* into this expression yields:

$$-\mu_k(-F\sin\phi + mg) + F\cos\phi = 0$$
$$\Rightarrow \quad F = [\mu_k mg] / [\mu_k \sin\phi + \cos\phi].$$

v.) Given that the body is moving with a constant velocity (i.e., it isn't accelerating), we now have a function that defines the *force applied* in terms of the *angle of the force*. Taking the derivative of that function and setting it equal to zero yields an expression from which the angle of minimum force can be determined. Using the Chain Rule to determine the expression, we get:

$$\frac{dF(\phi)}{d\phi} = \frac{d\left[\dfrac{\mu_k mg}{\mu_k(\sin\phi)+\cos\phi}\right]}{d\phi}$$

$$= \frac{d\left[\mu_k mg\left[\mu_k(\sin\phi)+\cos\phi\right]^{-1}\right]}{d\phi}$$

$$= \mu_k mg(-1)\left[\mu_k(\sin\phi)+\cos\phi\right]^{-2}\left[\mu_k(\cos\phi)-\sin\phi\right]$$

$$= \frac{-\mu_k mg\left[\mu_k(\cos\phi)-\sin\phi\right]}{\left[\mu_k(\sin\phi)+\cos\phi\right]^2}$$

vi.) As ungodly as this may look, the criterion for this expression equaling zero is relatively simple. All that must be true is that the numerator equal zero. That will be satisfied if $\mu_k(\cos\phi) - (\sin\phi) = 0$. With that observation:

$$\mu_k(\cos\ \phi) - (\sin\ \phi) = 0,$$
$$\Rightarrow \quad \mu_k = [\sin\ \phi]/[\cos\ \phi]\ .$$

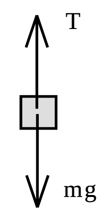
vii.) As *sin* (ϕ)/*cos* (ϕ) = *tan* ϕ, we can write:

$$\phi = \tan^{-1}(\mu_k).$$

This is the optimal angle at which the force *F* will be a minimum.

5.4)

a.) A stationary elevator will feel no friction; the f.b.d. for the situation is shown in the sketch to the right. Using N.S.L.:

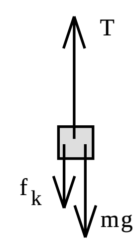


$\underline{\sum F_y:}$

$$\begin{aligned} T - mg &= ma \\ &= 0 \qquad \text{(as elevator's acc. } a_e = 0\text{)} \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad T &= mg \\ &= (400\text{ kg})(9.8\text{m/s}^2) \\ &= 3920\text{ nts.} \end{aligned}$$

b.) With the upward acceleration of the elevator, the frictional force will be applied downward as shown in the f.b.d. to the right. The acceleration term *a* is a magnitude whose sign (manually placed) is positive. N.S.L. yields:



$\underline{\sum F_y:}$

$$T - mg - f_k = +ma$$

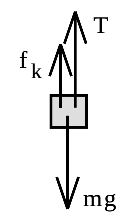
$$\begin{aligned} \Rightarrow \quad T &= mg + f_k + ma \\ &= (400\text{ kg})(9.8\text{m/s}^2)+(80\text{ nt})+(400\text{ kg})(2.8\text{ m/s}^2) \\ &= 5120\text{ nts.} \end{aligned}$$

c.) The only difference between this problem and *Part b* is that the acceleration is zero (constant velocity means zero acceleration). It makes no difference what the velocity actually is; the forces acting on the elevator are the same as in *Part b* so the f.b.d. from *Part b* is still valid. Using it, we get:

$\underline{\sum F_y:}$

$$T - mg - f_k = ma$$
$$\Rightarrow \quad T = mg + f_k + m(0)$$
$$\Rightarrow \quad = (400\ kg)(9.8 m/s^2) + (80\ nt)$$
$$= 4000\ nts.$$

d.) With the downward velocity, friction is upward as shown in the f.b.d. to the right. N.S.L. yields:



$\underline{\sum F_y:}$

$$T - mg + f_k = -ma$$
$$\Rightarrow \quad T = mg - f_k - ma$$
$$= (400\ kg)(9.8 m/s^2) - (80\ nt) - (400\ kg)(2.8\ m/s^2)$$
$$= 2720\ nts.$$

Note: Whenever you can, make the acceleration term *a* a magnitude. That is what I've done above (the acceleration's negative sign has been unembedded). Be careful when you do this, though. Don't put a negative sign in front of the *a*, then proceed to use ***-2.8 m/s²*** when it comes time to put in the numbers.

e.) Moving with a constant velocity means that the acceleration *a* is zero. Friction is still acting (upward in this case), so the f.b.d. used in *Part d* is still valid (the forces haven't changed, there is just no acceleration).

$\underline{\sum F_y:}$

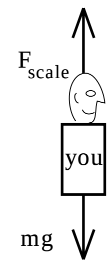
$$T - mg + f_k = -ma$$
$$\Rightarrow \quad T = mg - f_k - m(0)$$
$$= (400\ kg)(9.8 m/s^2) - (80\ nt)$$
$$= 3840\ nts.$$

5.5) The scale in this case is measuring the net force you apply to the scale (or the ground). If the acceleration is upward, this force F_{scale} will be *greater than mg*; if downward, it will be *less than mg*. To determine the acceleration direction, we need to determine *mg*:

$$mg = (60\ kg)(9.8\ m/s^2)$$
$$= 588\ newtons.$$

As this is less than the scale reading of 860 newtons, the elevator must be accelerating upward and the acceleration's sign must be *positive*.

By Newton's Third Law, the force you apply to the scale must be equal and opposite the force the scale applies to you. As such, using an f.b.d. and N.S.L. on yourself (see to right) yields:



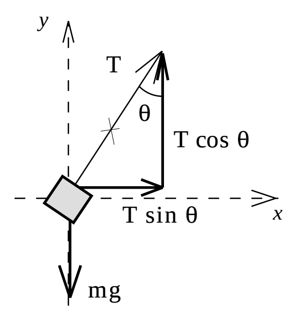
$\underline{\sum F_y:}$

$$F_{scale} - mg = ma$$
$$\Rightarrow \quad a = (F_{scale}/m) - g$$
$$= (860\ \text{nt})/(60\ \text{kg}) - (9.8\text{m/s}^2)$$
$$= 4.53\ \text{m/s}^2.$$

Note: If we had assumed a downward acceleration (i.e., an acceleration that was *negative*), we would have gotten a negative sign in front of the calculated *a* term above. The *negative sign* in an answer like that does not identify direction. By unembedding the sign, we have made the acceleration term a *magnitude*. As such, it should be positive. The negative sign in front of an answer in such instances means we have assumed the *wrong* direction for the acceleration, nothing else!

5.6)

a.) *An f.b.d.* for the forces on the mass is shown to the right. Noting that the acceleration is to the right, I have put one coordinate axis along the horizontal. N.S.L. in the *x direction* yields:



$\underline{\sum F_x:}$

$$T \sin\theta = ma$$
$$\Rightarrow \quad a = (T \sin\theta)/m \qquad \text{(Equation A)}.$$

We need to determine *T* to solve this. Using N.S.L. in the *y direction* yields:

$\underline{\sum F_y:}$

$$T \cos\theta - mg = ma_y$$
$$= 0 \qquad (\text{as } a_y = 0)$$

$$\Rightarrow \quad T = mg/(\cos\theta).$$

Re-writing, then substituting back into *Equation A* yields:

$$a = [T](\sin\theta)/m$$
$$= [mg/(\cos\theta)](\sin\theta)/m.$$

The *m's* cancel and $(\sin\theta)/(\cos\theta)$ is $\tan\theta$, so we end up with

$$a = g\tan\theta.$$

For our problem, the numbers yield:

$$a = (9.8\ \text{m/s}^2)(\tan 26^o)$$
$$= 4.78\ \text{m/s}^2.$$

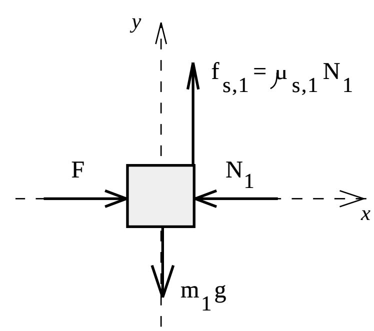
b.) At constant velocity, there is no acceleration and, hence, no swing observed. The string and mass should hang completely vertical. Note: That is exactly what the equation in the *x direction* suggests. The only time the acceleration will equal zero in $T\sin\theta = ma$ is when $\theta = 0$.

Note: One intrepid student whose father was a pilot pointed out that airplane floors (and ceilings) are not horizontal (she observed that when she walks to the bathroom at the rear of a plane, she always walks down hill). In any case, that idiosyncracy isn't important here as the angle is measured relative to the vertical.

5.7)

a.) We are interested in finding the *coefficient of static friction* between both m_1 and m_2 (call this $\mu_{s,1}$) and between m_2 and the wall (call this $\mu_{s,2}$), when *F = 25 newtons.*

--To the right is the f.b.d. for m_1. N.S.L. yields:



f.b.d. on mass m_1

$\underline{\sum F_x}$:

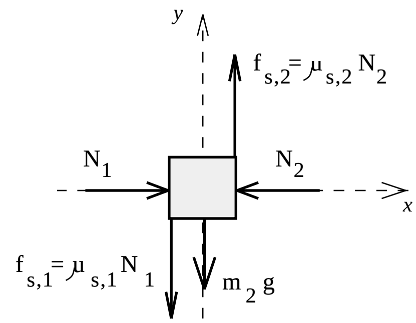
$$F - N_1 = m_1a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad N_1 = F \quad (\text{equal to 25 nts}).$$

$\underline{\sum F_y}$:

$$\mu_{s,1}N_1 - m_1g = m_1a_1$$
$$= 0 \qquad (\text{as } a_1 = 0).$$
$$\Rightarrow \quad \mu_{s,1} = (m_1g)/N_1$$

$= [(2\ \text{kg})(9.8\ \text{m/s}^2)] / (25\ \text{nt})$
$= .784$ (note that the coefficient is unitless).

--The f.b.d. for m_2 is shown to the right. A number of observations need to be made before dealing with N.S.L.:



f.b.d. on mass m_2

i.) Look at *m_1's* f.b.d. on the previous page. Notice that it experiences a *normal force N_1* due to its being jammed up against m_2. As such, m_2 must feel a *reaction force* (Newton's Third Law) of the same magnitude (i.e., N_1) in the *opposite direction*. That force has been placed on *m_2's* f.b.d.

ii.) Look again at *m_1's* f.b.d. on the previous page. Notice that it experiences a frictional force $f_{s,1}$ due to its rubbing up against m_2. As such, m_2 must feel a *reaction force* of magnitude $f_{s,1}$ in the direction opposite that of the frictional force on m_1. That force has been placed on *m_2's* f.b.d.

iii.) Having made those observations, N.S.L. yields:

$\underline{\sum F_x}$:

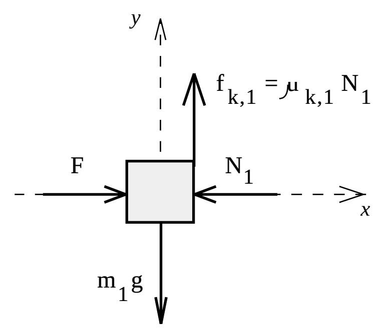
$$N_1 - N_2 = m_2 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad N_1 = N_2 \quad (\text{equal to } F = 25\ nts \text{ as } N_1 = F)\ .$$

$\underline{\sum F_y}$:

$$\mu_{s,2} N_2 - \mu_{s,1} N_1 - m_2 g = m_2 a_2$$
$$= 0 \quad (\text{as } a_2 = 0)$$
$$\Rightarrow \quad \mu_{s,2} = [\mu_{s,1} N_1 + m_2 g] / N_2$$
$$= [(.784)(25\ \text{nt}) + (7\ \text{kg})(9.8\ \text{m/s}^2)] / (25\ \text{nt})$$
$$= 3.528.$$

b.) The force F is now 20 newtons. That means there is not enough force associated with F to keep the bodies pinned to the wall. That being the case, they begin to accelerate downward. Assume the *coefficients of kinetic friction* are $\mu_{k,1} = .15$ and $\mu_{k,2} = .9$ respectively.

As innocuous as this scenario may seem, the problem has the potential to be a real stinker. Why? Because the direction of a frictional force on a body depends upon the direction of its slide *relative to the other body*. We don't know the *acceleration* of each of the bodies. We do know that if m_2 accelerates downward <u>faster</u> <u>than</u> m_1, then m_1*'s* motion *relative to* m_2 will be <u>upward</u> and the frictional force on m_1 will be *downward*. If m_2 accelerates downward <u>more slowly</u> than m_1, then m_1*'s* motion *relative to* m_2 will be <u>downward</u> and the frictional force on m_1 will be *upward*. Not knowing the acceleration of either body means we don't know which body will be moving faster and, hence, what direction the frictional force will be on either object. In short, we have to do the problem both ways to see which ends up making sense.



f.b.d. on mass m_1

We will start by assuming m_1 accelerates faster than m_2. In that case, the frictional force on m_1 will be upward and the f.b.d. for the situation will be as shown to the right. Using N.S.L. on m_1, we get:

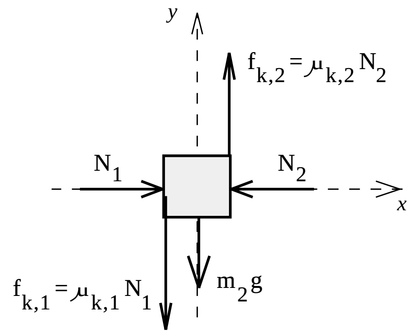
<u>$\sum F_x$:</u>

$$F - N_1 = m_1 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad F = N_1 \quad (\text{equal to 20 nt}).$$

<u>$\sum F_y$:</u>

$$\mu_{k,1} N_1 - m_1 g = -m_1 a_1.$$
$$\Rightarrow \quad a_1 = [-\mu_{k,1} N_1 + m_1 g]/m_1$$
$$= [-(.15)(20\text{ nt}) + (2\text{ kg})(9.8\text{ m/s}^2)] / (2\text{ kg})$$
$$= 8.3\text{ m/s}^2.$$

--The f.b.d. for the forces acting on m_2 are shown on the next page. N.S.L. yields:



f.b.d. on mass m_2

$\underline{\sum F_x}$:

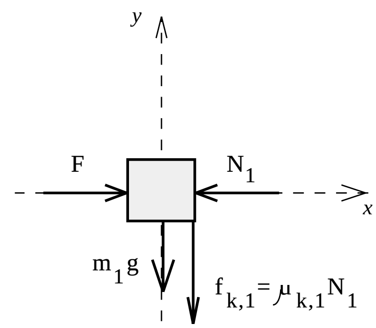
$$N_1 - N_2 = m_2 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad N_1 = N_2 \quad (\text{equal to } F = 20 \text{ nts}).$$

$\underline{\sum F_y}$:

$$\mu_{k,2} N_2 - \mu_{k,1} N_1 - m_2 g = -m_2 a_2$$
$$\Rightarrow \quad a_2 = [-\mu_{k,2} N_2 + \mu_{k,1} N_1 + m_2 g] / m_2$$
$$= [-(.9)(20 \text{ nt}) + (.15 \text{ kg})(20 \text{ nt}) + (7 \text{ kg})(9.8 \text{ m/s}^2)] / (7 \text{ kg})$$
$$= 7.66 \text{ m/s}^2.$$

Note 1: Yes! We've lucked out. We assumed m_1 accelerates faster than m_2, and that is just what our calculations have verified. If we had been wrong, we would have gotten senseless results. As we got it right on the first try, we needn't go further.

Note 2: For the amusement of it, let's go further. That is, assume that m_1 accelerates *more slowly* than m_2. That means m_1 will slide *upward* relative to m_2 and the frictional force will be downward (this is exactly opposite the situation we outlined above). With the direction of the frictional force reversed, the f.b.d. on m_1 look as shown to the right. N.S.L. yields:



f.b.d. on mass m_1

$\underline{\sum F_x}$:

$$F - N_1 = m_1 a_x$$
$$= 0 \quad (\text{as } a_x = 0)$$
$$\Rightarrow \quad F = N_1 \quad (= 20 \text{ nts}).$$

$\underline{\sum F_y}$:

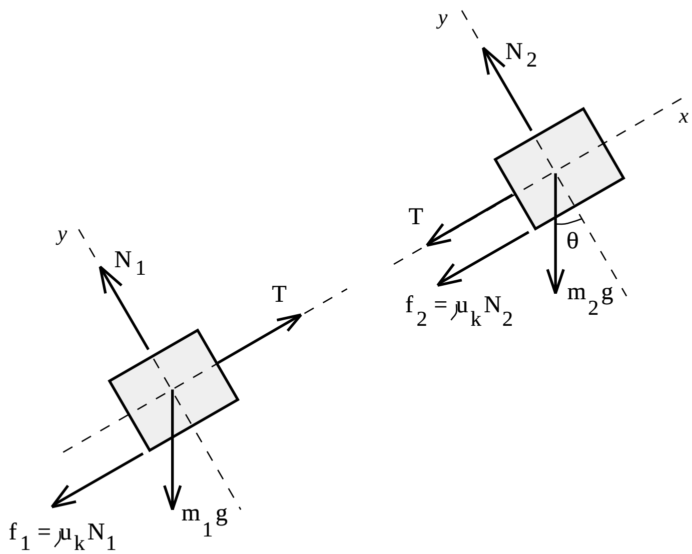
$$-\mu_{k,1} N_1 - m_1 g = -m_1 a_1.$$
$$\Rightarrow \quad a_1 = [\mu_{k,1} N_1 + m_1 g] / m_1$$
$$= [(.15)(20 \text{ nt}) + (2 \text{ kg})(9.8 \text{ m/s}^2)] / (2 \text{ kg})$$
$$= 11.3 \text{ m/s}^2.$$

Yikes! According to our calculations, block m_1 is accelerating faster than the *acceleration of gravity* ($g = 9.8\ m/s^2$). That isn't possible in this situation. Conclusion? We made bad assumptions about the acceleration of m_1 and m_2.

c.) The reason the accelerations are different? They have different forces acting on them!

5.8)

a.) The *free body diagram*s for this situation are shown to the right.



b.) We need the frictional forces in both cases, which means we need both N_1 and N_2. Using N.S.L. in the *y direction*:

$\underline{\sum F_y}$:

$$N_1 - m_1 g \cos\theta = m_1 a_y$$

$$\Rightarrow \quad N_1 = m_1 g \cos\theta \qquad (\text{as } a_y = 0).$$

Likewise, $N_2 = m_2 g\cos\theta$.

--Using N.S.L. for the *x-motion* of m_1, noting that the acceleration is in the *negative* direction, relative to our coordinate axis (the body is slowing, hence the acceleration is *opposite* the direction of the velocity):

$\underline{\sum F_x}$:

$$T - \mu_k N_1 - m_1 g \sin\theta = -m_1 a.$$

Substituting in for N_1 and solving for $m_1 a$, we get:

$$m_1 a = [-T + \mu_k(m_1 g \cos\theta) + m_1 g \sin\theta] \qquad \text{(Equation A)}.$$

--To get rid of the tension term, consider the *x motion* of m_2:

$\underline{\sum F_x}$:

$$-T - \mu_k N_2 - m_2 g \sin\theta = -m_2 a.$$

Substituting in for N_2 and solving for the tension T, we get:

$$T = -\mu_k(m_2 g \cos\theta) - m_2 g \sin\theta + m_2 a.$$

Substituting the tension term into *Equation A* yields:

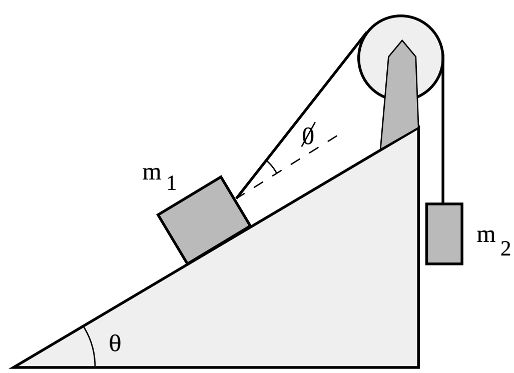
$$m_1 a = [-(-\mu_k(m_2 g \cos\theta) - m_2 g \sin\theta + m_2 a) + \mu_k(m_1 g \cos\theta) + m_1 g \sin\theta].$$

Solving for the acceleration yields:

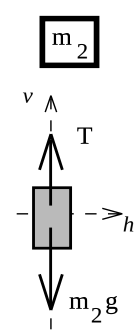
$$a = [\mu_k(m_2 g \cos\theta) + m_2 g \sin\theta + \mu_k(m_1 g \cos\theta) + m_1 g \sin\theta]/(m_1 + m_2)$$
$$= \mu_k g \cos\theta + g \sin\theta.$$

c.) Plugging the expression for *a* back into *Equation A* allows us to determine *T*. I'll save space by leaving the exercise to you.

5.9) This is an important situation because it requires you to face all the pitfalls that can occur when doing incline-plane problems.



We know m_1 is moving *down* the incline. That means m_2 is moving upward. Unfortunately, we have not been told the direction of *acceleration* for either m_1 or m_2. For the sake of amusement, let's assume m_1*'s* acceleration is *up* the incline (i.e., it's slowing). That will make m_2*'s* acceleration (remember, it's physically moving upward) *downward* (i.e., it's also slowing). Consider m_2*'s* f.b.d. first. N.S.L. allows us to write:

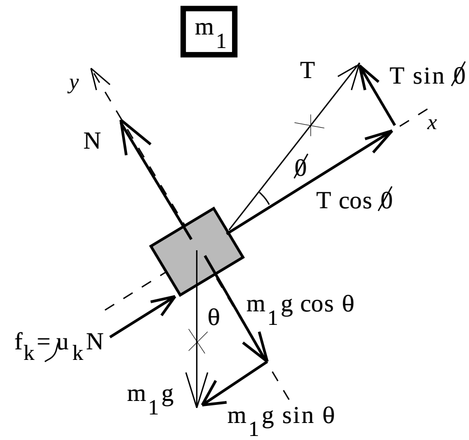


$\underline{\sum F_y}$:

$$T - m_2 g = -m_2 a_2$$

$$\Rightarrow \quad T = m_2 g - m_2 a_2 \qquad \text{(Equation 1).}$$

Remembering that the magnitude of m_1*'s* acceleration is numerically equal to $a_2 \cos\theta$ (this was pointed out in the original set-up), now consider m_1*'s* f.b.d. N.S.L. yields:



$\underline{\sum F_x}$:

$$T\cos\phi + \mu_k N - m_1 g\sin\theta = m_1 a_1$$
$$\Rightarrow \quad T\cos\phi + \mu_k N - m_1 g\sin\theta = m_1(a_2\cos\theta)$$
(Equation 2).

At this point, we have three unknowns *N*, *a*, and *T*. To determine an expression for *N*, consider N.S.L. in the *y direction* for m_1. Doing so yields:

$\underline{\sum F_y}$:

$$T\sin\phi + N - m_1 g\cos\theta = m_1 a_y = 0 \qquad (\text{as } a_y = 0)$$
$$\Rightarrow \quad N = -T\sin\phi + m_1 g\cos\theta \qquad (\text{Equation 3})$$

--Note that although the problem did not ask you to do so, solving for a_2 is done in the following manner.

Plugging *Equation 1* into *Equation 3* yields:

$$N = -(m_2 g - m_2 a_2)\sin\phi + m_1 g\cos\theta \qquad (\text{Equation 4})$$

Plugging *Equation 1* and *Equation 4* into *Equation 2* yields:

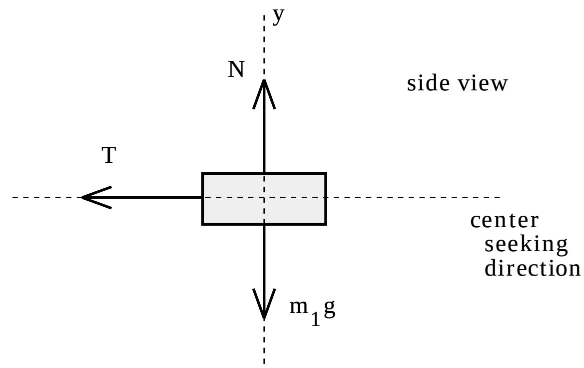
$$\underbrace{(m_2 g - m_2 a_2)}_{T}\cos\phi + \mu_k\underbrace{[-(m_2 g - m_2 a_2)\sin\phi + m_1 g\cos\theta}_{N} - m_1 g\sin\theta = m_1(a_2\cos\phi)$$

Rearranging and solving for a_2 yields:

$$a_2 = \frac{m_2 g\cos\phi - \mu_k m_2 g\sin\phi + \mu_k m_1 g\cos\theta - m_1 g\sin\theta}{m_1\cos\phi + m_2\cos\phi - \mu_k m_2\sin\phi}.$$

Interesting Note: There are positive and negative parts of the denominator, but it's OK because the two amounts will never add to zero.

5.10) This is a circular motion problem. There must be a natural force somewhere in the system that acts to change the direction of *m_1's* motion. That is, there must be a *gravitational* or *normal* or *tension* or *friction* or *push-me-pull-you* force that is *center-seeking*. In this case, that force provided by the system is the tension in the string. The problem proceeds:



Using N.S.L. on mass m_1 (see f.b.d. to right):

$$\underline{\sum F_c:}$$

$$-T = -m_1 a_c$$

$$\Rightarrow \quad T = m_1\,(v^2/R)$$

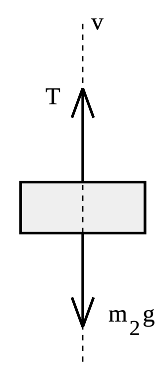
$$\Rightarrow \quad v = (TR/m_1)^{1/2}.$$

This equation has two unknowns, v and T. To get rid of the tension term, consider N.S.L. applied to mass m_2 (see f.b.d. to right):

$$\underline{\sum F_v:}$$

$$T - m_2 g = 0 \qquad (\text{as } a_y = 0)$$

$$\Rightarrow \quad T = m_2 g.$$



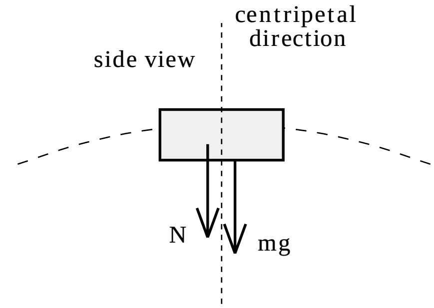
Substituting back into our expression for v, we get:

$$v = [TR/m_1]^{1/2}$$

$$= [(m_2 g)R/m_1]^{1/2}.$$

This is a nice problem as it requires you to deal with more than one body. The approach is the same as it has always been. Do *an f.b.d.* for one body in the system. In this case, notice that the body is moving in a circular path. As such, orient one axis so that it is *center-seeking* (i.e., along the radius of the arc upon which the bob is moving). Use N.S.L. to generate as many equations as needed. If you haven't enough equations to solve for the desired unknown, pick a second mass and repeat the approach.

5.11) *An f.b.d.* for the forces acting on the cart when at the top of the loop is shown to the right. N.S.L. yields:



$$\underline{\sum F_c}:$$

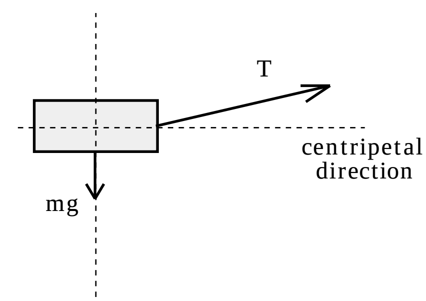
$$-N - mg = -m\,a_c$$
$$= -m\,(v^2/R)$$
$$\Rightarrow \quad v = [(N + mg)R/m]^{1/2}.$$

When the cart just freefalls through the top of the arc, the normal force goes to zero. In that case:

$$v = [gR]^{1/2}.$$

5.12)

a.) To begin with, the tension vector must have a *vertical component* (see f.b.d. to the right). If it doesn't, there will be nothing to counteract gravity and the rock must accelerate downward--something our object is not doing. As such, that vertical force will ALWAYS equal *mg*. BUT, if the rock is moving fast, the angle will be small and the vertical component will be very small *in comparison to T*. In that case, we can assume the tension force T is *wholly centripetal* and $r = L$. Using those assumptions:



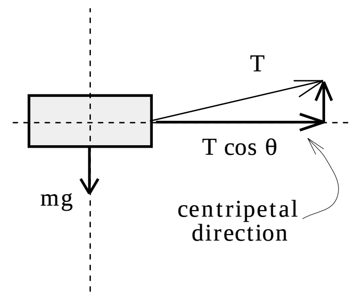
$$\underline{\sum F_c}:$$

$$T = m\,a_c$$
$$= m\,(v^2/L)$$
$$\Rightarrow \quad v = [TL/m]^{1/2}.$$

Putting in the numbers and using T_{max}, this yields:

$$v = [TL/m]^{1/2}$$
$$= [(50\ \text{nt})(1.2\ \text{m})/(.2\ \text{kg})]^{1/2}$$
$$= 17.32\ \text{m/s}.$$

b) Because there is centripetal motion going on here, the temptation is to draw *an f.b.d.* like the one shown to the right and then sum the forces in the *center-seeking direction*. Noting that the radius *r* of the body's motion is $L \cos \theta$, we write:



$\underline{\sum F_c}$:

$$T \cos \theta = m a_c$$
$$= m [v^2/r]$$
$$= m [v^2/(L \cos \theta)]$$
$$\Rightarrow (\cos \theta)^2 = [mv^2/LT].$$

This equation would be great if we knew the velocity and wanted the angle (or vice versa). Unfortunately, we know neither. In other words, for this particular question, summing in the *center-seeking direction* is going to be no help at all (at least not initially). With that in mind, let's use N.S.L. in the *vertical direction* and pray it gives us an equation we can use.

$\underline{\sum F_v}$:

$$T \sin \theta - mg = 0 \quad (\text{as } a_y = 0)$$
$$\Rightarrow \sin \theta = mg / T_{max}$$
$$= (.2 \text{ kg})(9.8 \text{ m/s}^2) / (50 \text{ nt})$$
$$= .039$$
$$\Rightarrow \theta = 2.247^o.$$

c.) We now know the angle that corresponds to the velocity at which the string will give up and break. With that information we can use N.S.L. in the *center-seeking* direction to bring the velocity term into play (that equation was derived above--it is re-derived below for your convenience). Doing so yields:

$\underline{\sum F_c}$:

$$T \cos \theta = m a_c$$
$$= m [v^2/(L \cos \theta)]$$
$$\Rightarrow v = [LT(\cos \theta)^2/m]^{1/2}$$
$$= [(1.2 \text{ m}) (50 \text{ nt}) (\cos 2.247^o)^2 / (.2 \text{ kg})]^{1/2}$$
$$= 17.3 \text{ m/s}.$$

Notice how close this is to the solution determined in *Part a*. The reason for this should be obvious. The string-breaking velocity is high which means the string-breaking angle is small. Being so, the vertical tension component (this must equal *mg*) will be small in comparison to the overall tension *T* and the *horizontal tension component* will very nearly equal *T*. The assumption we made in *Part a* was that the tension was all in the center-seeking (horizontal) direction--in this case, that wasn't a bad assumption to make.

d.) For this part, we must incorporate the velocity into our analysis (we didn't do that when we were looking for the angle in *Part b*; you should understand the difference between these two situations). Using the f.b.d. shown in *Part b-i*, we can use N.S.L. to write:

$\underline{\sum F_c}$:

$$\begin{aligned} T\cos\theta &= m\,a_c \\ &= m\,[v^2/(L\cos\theta)] \\ \Rightarrow\quad v &= [TL(\cos\theta)^2/m]^{1/2} \qquad \text{(Equation A).} \end{aligned}$$

In this case, we don't know *T*. Looking at the vertical forces yields:

$\underline{\sum F_v}$:

$$\begin{aligned} T\sin\theta - mg = 0 \qquad &(\text{as } a_y = 0) \\ \Rightarrow\quad T = mg/\sin\theta. & \end{aligned}$$

Substituting *T* into Equation A:

$$\begin{aligned} v &= [T(\cos\theta)^2 L/m]^{1/2} \\ &= [(mg/\sin\theta)\,(\cos\theta)^2 L/m]^{1/2} \\ &= [(g/\sin\theta)\,(\cos\theta)^2 L]^{1/2} \\ &= [g\,(\cot\theta)\,(\cos\theta)L]^{1/2}. \end{aligned}$$

NOTE: If you don't like the use of the cotangent function (cos/sin), forget it and simply use the sine and cosine terms as presented.

Putting in the numbers, we get:

$$\begin{aligned} v &= [(9.8\text{ m/s}^2)(\cot 30^\circ)(\cos 30^\circ)(1.2\text{ m})]^{1/2} \\ &= 4.2\text{ m/s}. \end{aligned}$$

5.13)

a.) The gravitational force between you and the earth, using Newton's general gravitational expression, is:

$$\begin{aligned} F_g &= G\, m_{you} m_e / r^2 \\ &= (6.67\text{x}10^{-11}\ m^3/kg\cdot s^2)\ (70\ kg)\ (5.98\text{x}10^{24}\ kg)\ /\ (6.37\text{x}10^6\ m)^2 \\ &= 688\ \text{nts.} \end{aligned}$$

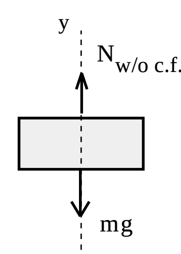
Using $m_{you}g$:

$$\begin{aligned} F_g &= m_{you} g \\ &= (70\ kg)\ (9.8\ m/s^2) \\ &= 686\ \text{nts.} \end{aligned}$$

The discrepancy is due to round-off error.

Note: The reason we can get away with using *mg* when near the earth's surface is due to the fact that the earth's radius is so large. That is, it really doesn't matter whether you are on the earth's surface or 200 meters above the earth's surface. For all intents and purposes, the distance between you and the center of the earth is going to be, to a very good approximation, the same.

b.) Let's begin by determining the amount of normal force ($N_{w/o\ c.f.}$) the earth must apply to you *when you stand at the poles*. The f.b.d. for the situation is shown to the right. Noting that there is no centripetal acceleration at the poles (at the poles the rotational speed of the earth is zero), a_y is zero and N.S.L. yields:



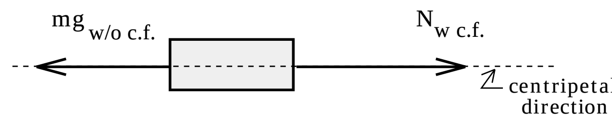
$\underline{\sum F_y:}$

$$\begin{aligned} N_{w/o\ c.f.} - mg_{w/o\ c.f.} &= 0 \qquad (\text{as } a_y = 0) \\ \Rightarrow\quad N_{w/o\ c.f.} &= mg_{w/o\ c.f.} \\ &= (70\ kg)(9.83\ m/s^2) \\ &= 688.1\ \text{newtons.} \end{aligned}$$

Note that at the equator, the earth's rotational speed is equal to *the distance a point on the equator travels in one day* (i.e., the circumference = $2\pi R = (2)(3.14)(6.37\text{x}10^6\ m) = 4\text{x}10^7\ m$) divided by *the time it takes to do the traveling* (i.e., *24 hours = 86,400 seconds*), or:

$$\begin{aligned} v_{eq} &= d / t \\ &= (4\text{x}10^7 \text{ m}) / (86{,}400 \text{ sec}) \\ &= 463.2 \text{ m/s} \qquad\qquad \text{(this is around 1000 mph).} \end{aligned}$$

Let's now determine the amount of normal force ($N_{w\ c.f.}$) the earth must apply to you *when you stand at the equator*. The f.b.d. for the situation is shown to the right. Noting that as there is centripetal acceleration at the equator (at the equator there is rotational speed in the amount calculated above), a_y is non-zero and N.S.L. yields:



$\underline{\sum F_c:}$

$$\begin{aligned} N_{w\ c.f.} - mg_{w/o\ c.f.} &= -ma_c \\ &= -m(v^2/R) \\ \Rightarrow \quad N_{w\ c.f.} &= mg_{w/o\ c.f.} - m(v^2/R). \end{aligned}$$

Put in a different context, the normal force required at the equator will be equal to the normal force required without centripetal force (remember, $N_{w/o\ c.f.} = mg_{w/o\ c.f.}$ from above) minus the centripetal force (this will numerically equal mv^2/R) required to move you into circular motion. Putting in the numbers yields:

$$\begin{aligned} N_{w\ c.f.} &= mg_{w/o\ c.f.} - m(v^2/R) \\ &= (688.1 \text{ nts}) - (70 \text{ kg})[(463.2 \text{ m/s})^2/(6.37\text{x}10^6 \text{ m}) \\ &= 685.7 \text{ nts.} \end{aligned}$$

If we wanted to define a gravitational constant g_{equ} that, when multiplied by your mass gives the amount of force the earth must exert on you when you stand at the equator (that is exactly how the *g* value you have come to know and love was originally determined), g_{equ} will be:

$$\begin{aligned} N_{w\ c.f.} &= mg_{w\ c.f.} \\ \Rightarrow \quad g_{w\ c.f} &= N_{w\ c.f.}/m \\ &= (685.7 \text{ nts}) / (70 \text{ kg}) \\ &= 9.796 \text{ m/s}^2. \end{aligned}$$

c.) Defining the distance between the earth and moon to be *r* and using N.S.L., we get:

$\underline{\sum F_c}$:

$$-G\, m_e m_m / r^2 = -m_m\, a_c$$
$$= -m_m\, v^2/r$$
$$\Rightarrow \quad v = (Gm_e/r)^{1/2} \qquad \text{(Equ. A)}$$
$$= [(6.67\text{x}10^{-11}\ m^3/kg{\cdot}s^2)\ (5.98\text{x}10^{24}\ kg)\ /\ (3.84\text{x}10^8\ m)]^{1/2}$$
$$= 1019\ m/s.$$

Interesting Note: Just as two objects will accelerate at the same rate (assuming neither gets close to its terminal velocity) under the influence of gravity, the velocity required to pull a mass in a given-radius circular path does NOT depend upon the mass of the object being so motivated. This might not be immediately obvious (just as the first statement in this NOTE wasn't obvious back when you first ran into it), but it is supported by the math. The moon is the mass being centripetally accelerated, and the m_m terms do cancel out in our velocity equation as derived above.